TSSTS 2020 Physics Division

TP301: Introduction to Turbulence

Guide to Course

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Introduction

"It is not the problem of finding new fundamental particles, but something left over from a long time ago—over a hundred years. Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of circulating or turbulent fluids."

— Richard P. Feynman [2]

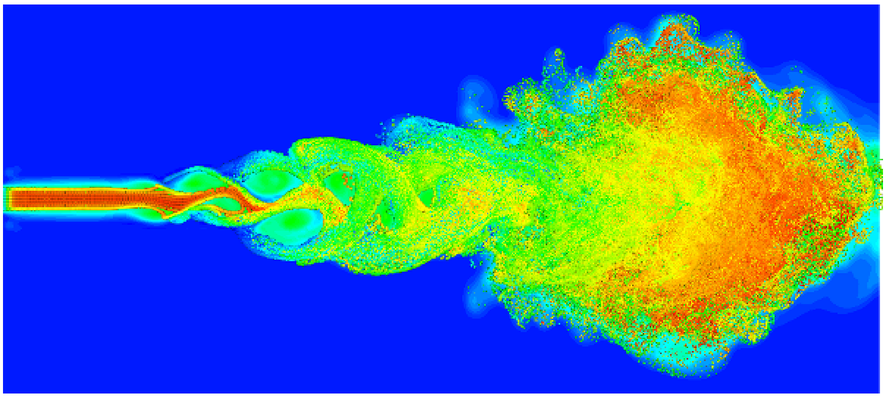

Figure 1: Turbulent Jet

Described by Feynman (and perhaps originating from Einstein) as the most important unsolved problem in classical physics/fluid mechanics, turbulence remains only partly understood despite having attracted the sustained efforts of many leading scientists for well over a century [1]. As an example, Heisenberg did his PhD on turbulence with Sommerfeld, from which he decided to move on to quantum mechanics.

In this course, I will introduce to you the physics and mathematics of turbulent flow. The course content is based on the *Introduction to Turbulence and Modelling* course I took from Prof. J. C. Vassilicos & Dr. O. Buxton during the Autumn term in 2018 and the *Hydrodynamic Stability* course I took from my supervisor Dr. Yongyun Hwang during summer in 2019, both at Imperial [5] [3]. The book by [4] (2000) will also used throughout the course [4].

The expected duration of the lecture is 24h, in terms of 12 two-hours lectures. The exact timing is subject to change.

Course Outline

The equations of fluid motion

Continuum hypothesis. Eulerian and Lagrangian description. The continuity equation and the moment equation. Navier-Stoke equation. Vorticity transport equation. Dimensional analysis and Reynolds number.

Turbulence: guess guess who I am

Essential properties of turbulence. Classification of turbulence. Origin of turbulence.

Statistical fluid mechanics

Reynolds-Averaged Navier-Stokes equation. Ergodicity. The closure problem. Mean-flow equations.

Kolmogorov turbulence

One-point energy balance. Two-point energy balance. The energy cascade. Kolmogorov turbulence. Spectral analysis.

Wall turbulence

The logarithmic law of wall. Turbulent boundary layer. Turbulence structure.

Free shear turbulence

Equations for round jets. Self-simialr flows.

Hydrodynamic stability

Linearised Navier-Stokes equation. Inviscid theory - Rayleigh equation, Rayleigh inflection point criterion and Fjørtoft's theorem. Viscous theory - Orr-Sommerfeld equation and Squire equation, Squire's theorem.

Introduction to dynamical systems approach to turbulence

Galerkin projection - from PDE to ODE. Chaotic attractors and bifurcations. Landau's route to turbulence.

Prerequisites

Knowledge in differential equations, linear algebra and multivariable calculus is necessary. Previous experience in fluid mechanics will be helpful.

Disclaimer

Due to my background as an engineering student, there must exists some non-rigorous aspect of mathematical derivation. Question such as "the convergence of such and such intergal" will therefore be referred to related text books recommended during lecture.

References

[1] Davidson, P. A., Kaneda, Y., Moffatt, K., and Sreenivasan J, R. *A Voyage Through Turbulence.* Cambridge University Press, 2011.

[2] Feynman, R., Leighton, R., Sands, M., and Hafner, E. *The Feynman Lectures on Physics; Vol. I.* AAPT, 1965.

[3] Hwang, Y. *Hydrodynamic Stability.* Department of Aeronautics, Imperial College London, 2019.

[4] Pope, S. *Turbulent Flows.* Cambridge University Press, 2000.

[5] Vassilicos, J. C. *A Brief Introduction to Turbulence.* Department of Aeronautics, Imperial College London, 2018.