TSSTS 2020 Physics Division

TP201: Introduction to General Relativity

Guide to Course

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Introduction

Newtonian mechanics is useful for our everyday life, the most common motions we see around us are low in velocity (compared with the speed of light) and occur in a rather weak and approximately uniform gravitational field. But, generally, is it good enough to give us the right physics picture and understanding? The answer is no. Problems arose everywhere: Maxwell's equations suggests that the speed of light in vacuum is independent of observers, which is not compatible with Newtonian mechanics where velocity transformations for different observers are just simple addition/subtraction; Michelson-Morley experiment (1887) supported the above prediction, disproving Newtonian mechanics. This leads to the formulation of Special Relativity. But, when considering (Newtonian) gravity within the framework of SR, it failed since the gravitation in Newtonian picture is assumed as an instantaneous action at a distance which is not allowed in SR (no speed can exceed the speed of light). Also, Newtonian gravitation itself has flaws such as its failure in predict/explain the perihelion precession of Mercury and the bending of light. Thus, to incorporate SR and gravitation, General Relativity was born. I would say it is one of the greatest achievement of human mind and human civilisation obviously by its elegance and accuracy.

This course will be based on the *Introduction to General Relativity for Enthusiasts* course I taught at Centre for Mathematical Sciences, Cambridge during Michaelmas term 2019, and the course materials are far from original (mostly inspired by the *Relativity* course taught by Prof. A. Challinor at Cavendish Laboratory, University of Cambridge).

The whole lecture series will be roughly 24 hours, in terms of 12 two-hour lectures. The exact timing can be flexible regarding the actual circumstances.

Synopsis

Special Relativity: Bottom-Up

Inertial frames. Review of Newtionian picture of spacetime. Motivation of SR. Postulates of SR. Thought experiments: time dilation, length contraction, Lorentz transformation.

Spacetime perspective. Spacetime diagrams and lightcone structure. Velocity addition. Acceleration in SR.

Manifolds and Tensors

Manifolds. Functions and curves. Vectors and vector fields. Covectors and 1-forms. Tensors. Tensor operations.

Metric and Connection

Metric tensor. Benefits of having a metric: natural isomorphism, scalar product, length and volume. Normal coordinates. Connection. Levi-Civita connection. Useful concepts.

Parallel Transport and Geodesics

Parallel transport. Euler-Lagrange equations. Geodesics.

Special Relativity: Top-Down

Lorentz transformation. Relativistic dynamics: massive particles, massless particles. Local reference frame. Sniff of GR.

Invitation to a General Theory of Relativity

Problems with Newtonian gravity. The equivalence principle. Gravity as spacetime curvature.

Curved Spacetime

Intrinsic curvature. Riemann tensor. Ricci tensor and Ricci scaler. Geodesic deviation. Physical laws in curved spacetime.

Gravitational Field Equations

Energy-momentum tensor. Dust. Perfect fluids. Einstein equations. Weak field limit. Cosmological constant.

Schwarzschild Solution

Static isotropic metrics. Solution of vacuum Einstein equations. Birkhoff's theorem. Gravitational redshift. Particle trajectories.

Classical Tests of GR

Precession of planetary orbits. Bending of light.

Schwarzschild Black Holes

Singularities. Causal structure. Eddington-Finkelstein coordinates. Formation of black holes.

A Brief Introduction to Cosmology

Cosmological principle. Robertson-Walker metric. Expanding Universe. Cosmological field equations.

Linearised Gravity and Gravitational Waves

Weak field metric. Linearised field equations. Lorenz gauge. Wave solutions of linearised field equations.

Prerequisites

Working knowledge of multivariable calculus and some linear algebra is essential. Experience with solving differential equations is useful.

Bibliography

1. S. Carroll, *Lecture Notes on General Relativity*
   `https://arxiv.org/abs/gr-qc/9712019`

2. M.P. Hobson, G.P. Efstathion, A.N. Lasenby, *General Relativity: An Introduction for Physicists*

3. R. Wald, *General Relativity*