TSSTS 2020 Mathematics Division

MA666: Topology

Guide to Course

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June 22, 2020

Introduction

This course aims to provide an introduction to point-set topology, namely the study of the generalised notion of *continuity* in topological spaces. It is a natural continuation from results in analysis like the *Intermediate Value Theorem* and the *Heine-Borel Theorem*. We will rigorously define concepts like *neighborhood, closure, connected, compact, etc.*, and use them to prove some basic results such as $\mathbb{R}^2$ *is not homeomorphic to* $\mathbb{R}$. In the last part of the course, we will introduce the idea of a simplicial complex and prove some classical results regarding the classification of surfaces.

This course does not require any previous experiences with topology, but some familiarity with mathematical analysis would be helpful. The expected duration is 12 hours, but it could vary depending on the progress of the course.

Synopsis

Topological Space

Definition of a topological space. Open and closed sets. Limit Points. Closure and interior.

Continuity

Definition of continuity. Conditions for Continuity. Homeomorphisms.

Separation axioms

Hausdorff Space. Metric Space.

Different topologies

Subspace topology. Basis for a topology. Product topology.

Connectedness

Definition of connectedness. Continuous map of connected sets. Path connectedness.

Compactness

Definition of compactness. Continuous map of compact sets (In Hausdorff space). Sequential compactness.

Quotient Spaces and simplicial complex

Definition of quotient topology. Open maps. Construction of a simplicial complex.

Surfaces

Definition of a surface. Building surfaces by identifying sides of a polygon. Classifying different combinatorial surfaces.

References

[1] C. Drutu, M. Lackenby, *Notes on Part A Topology*, Oxford Maths
[2] James R. Munkres, *Topology* (2nd Ed.)